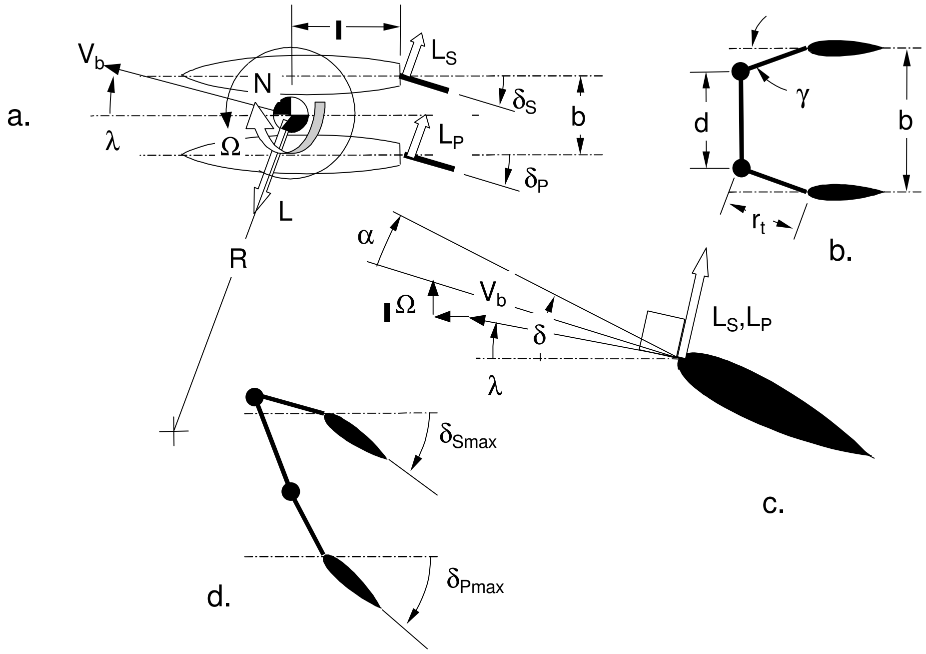
Ackermann Steering For Catamarans
Tom Speer



Figure

a.	boat geometry
b.	steering geometry
c.	local flow velocities at rudder
d.	extreme steering geometry

Symbol

b	beam
L	lift from hull & keel
L_P	lift from port rudder
L_S	lift from starboard rudder
l	distance from c.g. to rudder
N	yawing moment
R	turn radius
V_b	boat speed
α	angle of attack of rudder
δ	rudder deflection
δ_P	port rudder
δ_S	starboard rudder
λ	leeway angle
Ω	turn rate

<u>Assumptions</u>

1. Boat is turning at constant speed under power
2. Thrust = drag (approximately) so both can be neglected
3. Aerodynamic forces are ignored
4. Hull and rudders are not stalled, so hydrodynamic forces are operating in the linear range
5. Objective is to turn as tightly as possible

Turning Cat Geometry Figure a:

Boat is traveling at speed V_b, turning with radius R and turn rate Ω. It is making leeway (λ) as it crabs into the turn. The lift on the keel and hull, L, acts at right angles to the velocity (by definition), and this lift supplies the centripetal acceleration necessary to make the boat turn. The starboard and port rudders are deflected through the angles δ_S and δ_P. The rudders are located a distance l aft of the boat's center of gravity, and because this is a catamaran, the distance between them is the centerline beam, b.

The speed, turn radius, and turn rate are related by

1. $V_b = R \cdot \Omega$ $\quad R = \frac{V_b}{\Omega}$ $\quad \Omega = \frac{V_b}{R}$

The hydrodynamic lift from the hull is related to the speed by

2. $L = C_L \cdot \frac{1}{2} \cdot \rho \cdot V_b^2 \cdot S$

where C_L is the lift coefficient of the hull, ρ is the water density and S is the reference lateral area of the hull. The lift coefficient, C_L, is assumed to be a linear function of the leeway angle, λ:

3. $C_L = C_{L\lambda} \cdot \lambda$

The lift curve slope, $C_{L\lambda}$, is a characteristic of the hull and keel/board. Assuming the boat is turning as hard as it can,

4. $C_{Lmax} = C_{L\lambda} \cdot \lambda_{max}$

5. $L = C_{L\lambda} \cdot \lambda_{max} \cdot \frac{1}{2} \cdot \rho \cdot V_b^2 \cdot S = \left(C_{L\lambda} \cdot \lambda_{max} \cdot \frac{1}{2} \cdot \rho \cdot S\right) \cdot V_b^2$

λ_{max} is just short of the hull stalling, so this is another fixed characteristic of the boat, as is everything else in the parentheses.

The rudders produce lift forces of their own, and there is also a yawing moment, N, due to rudder forces, leeway, and turn rate which must be balanced. More on this later.

Linked Rudders Figure b:

The tillers of the two rudders are offset from the rudder centerline by the Ackermann angle, γ, so the link between them, whose length is d, is shorter than the beam. The length of the tillers is r_t. These quantities are related by:

6. $d = b - 2 \cdot r_t \cdot \sin(\gamma)$

7. $\frac{d}{b} = 1 - 2 \cdot \frac{r_t}{b} \cdot \sin(\gamma)$

$\delta_P := 0, 10 .. 70$

Since the rudders are linked together, once one rudder deflection is known, the other rudder can be determined as well. The beam of the boat, the two tillers and the link form a four-bar linkage - a very common mechanical arrangement. Let the coordinate system be positive X forward and positive Y to starboard. The locations of the ends of the tillers are

8. $X_P = r_t \cdot \cos\left(\delta_P + \gamma\right) - l$ $\qquad$ 9. $Y_P = r_t \cdot \sin\left(\delta_P + \gamma\right) - \frac{b}{2}$

10. $X_S = r_t \cdot \cos(\delta_S - \gamma) - 1$ 11. $Y_S = r_t \cdot \sin(\delta_S - \gamma) + \frac{b}{2}$

And the link ensures that the distance between the ends of the tillers remains a constant:

12. $d^2 = (X_S - X_P)^2 + (Y_P - Y_S)^2$

Substituting for the coordinates of the tillers gives the desired relationship of one rudder to the other and the basic constraint on the tiller motion:

13. $(b - 2 \cdot r_t \cdot \sin(\gamma))^2 = (r_t \cdot \cos(\delta_S - \gamma) - r_t \cdot \cos(\delta_P + \gamma))^2 + (r_t \cdot \sin(\delta_S - \gamma) + b - r_t \cdot \sin(\delta_P + \gamma))^2$

14.
$$
\begin{aligned}
0 = & \left[-2 \cdot \left(\cos(\delta_S - \gamma) \cdot \cos(\delta_P + \gamma) + 1 + \sin(\delta_S - \gamma) \cdot \sin(\delta_P + \gamma) - 2 \cdot \cos(\gamma)^2\right)\right] \cdot \frac{r_t}{b} \cdots \\
& + -2 \cdot \left(-\sin(\delta_S - \gamma) + \sin(\delta_P + \gamma) - 2 \cdot \sin(\gamma)\right)
\end{aligned}
$$

Once one rudder position is known, the other can be determined. For example, solving for the starboard rudder as a function of the port rudder position yields the following expression:

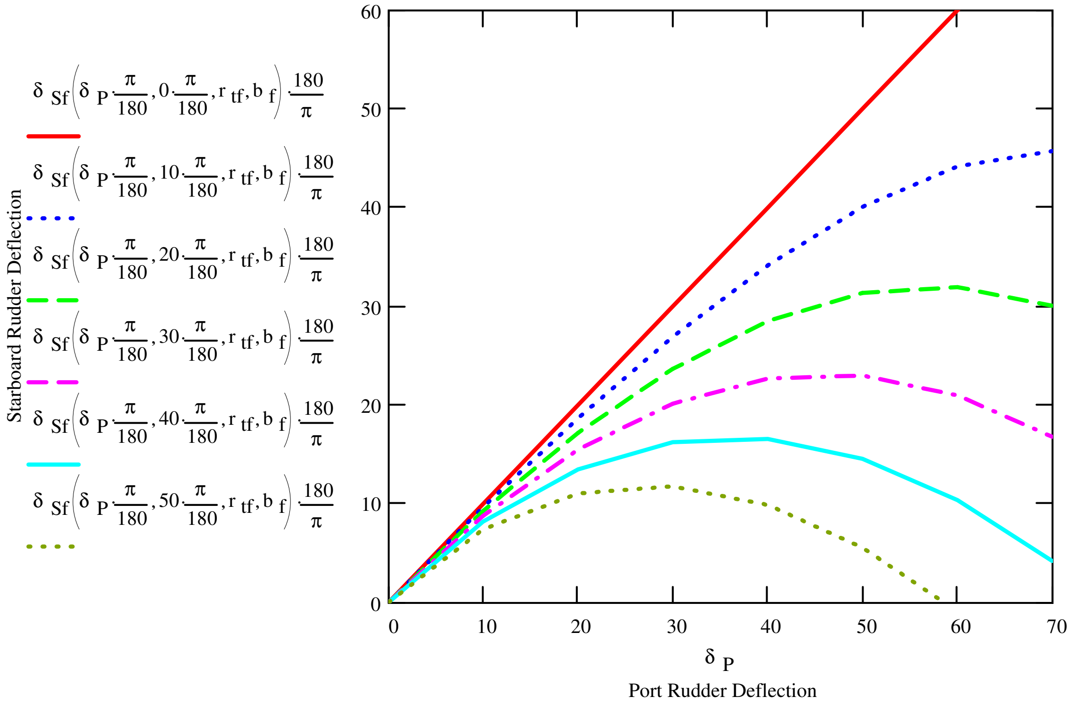
15.
$$
\delta_{Sf}(\delta_P, \gamma, r_t, b) := 2 \cdot \operatorname{atan}\left[\frac{
\left[\begin{aligned}
& 2 \cdot \cos(\gamma) + -2 \cdot \frac{r_t}{b} \cdot \left(\begin{aligned} & -\sin(\delta_P) + 2 \cdot \cos(\gamma)^2 \cdot \sin(\delta_P) \cdots \\ & + 2 \cdot \sin(\gamma) \cdot \cos(\delta_P) \cdot \cos(\gamma) \end{aligned}\right) \cdots \\
& + -2 \cdot \sqrt{\begin{aligned}
& \left[\begin{aligned} & \left(4 \cdot \sin(\gamma)^2 \cdot \cos(\delta_P) - \sin(\gamma)^2 \cdot \cos(\delta_P)^2\right) + \cos(\gamma)^2 \cdots \\ & + -3 \cdot \sin(\gamma)^2 - 2 \cdot \sin(\delta_P) \cdot \cos(\gamma) \cdot \cos(\delta_P) \cdot \sin(\gamma) \cdots \\ & + -\sin(\delta_P)^2 \cdot \cos(\gamma)^2 + 4 \cdot \cos(\gamma) \cdot \sin(\delta_P) \cdot \sin(\gamma) \end{aligned}\right] \cdots \\
& + \left[\begin{aligned} & -4 \cdot \cos(\delta_P) \cdot \sin(\gamma) - 8 \cdot \sin(\gamma) \cdot \cos(\gamma)^2 \cdots \\ & + 4 \cdot \cos(\gamma)^2 \cdot \sin(\gamma) \cdot \cos(\delta_P) + 4 \cdot \sin(\gamma) \cdots \\ & + -4 \cdot \sin(\gamma)^2 \cdot \cos(\gamma) \cdot \sin(\delta_P) \end{aligned}\right] \cdot \frac{r_t}{b} \cdots \\
& + \left[\begin{aligned} & \sin(\delta_P)^2 + 4 \cdot \cos(\gamma)^4 \cdot \cos(\delta_P)^2 \cdots \\ & + 4 \cdot \cos(\gamma)^2 \cdot \sin(\delta_P)^2 \cdot \sin(\gamma)^2 \cdots \\ & + -4 \cdot \sin(\delta_P)^2 \cdot \cos(\gamma)^2 \cdots \\ & + 4 \cdot \cos(\gamma)^2 + 4 \cdot \sin(\gamma)^2 \cdot \cos(\delta_P)^2 \cdot \cos(\gamma)^2 \cdots \\ & + -1 - 4 \cdot \cos(\gamma)^4 - 4 \cdot \cos(\delta_P)^2 \cdot \cos(\gamma)^2 \cdots \\ & + \cos(\delta_P)^2 + 4 \cdot \cos(\gamma)^4 \cdot \sin(\delta_P)^2 \end{aligned}\right] \cdot \left(\frac{r_t}{b}\right)^2
\end{aligned}}
\end{aligned}\right]
}{
\left[\begin{aligned}
& 2 \cdot \left(-3 \cdot \sin(\gamma) + \sin(\delta_P) \cdot \cos(\gamma) + \cos(\delta_P) \cdot \sin(\gamma)\right) \cdots \\
& + 2 \cdot \frac{r_t}{b} \cdot \left[\begin{aligned} & \left(1 + \cos(\delta_P)\right) - 2 \cdot \cos(\gamma)^2 \cdots \\ & + 2 \cdot \cos(\gamma) \cdot \sin(\delta_P) \cdot \sin(\gamma) - 2 \cdot \cos(\gamma)^2 \cdot \cos(\delta_P) \end{aligned}\right]
\end{aligned}\right]
}\right]
$$

This function is plotted below for the same dimensions Martin Schoon used and Ackermann angles ranging from 0 to 50 degrees. As expected, the starboard rudder always deflects less than the port rudder when turning to port.

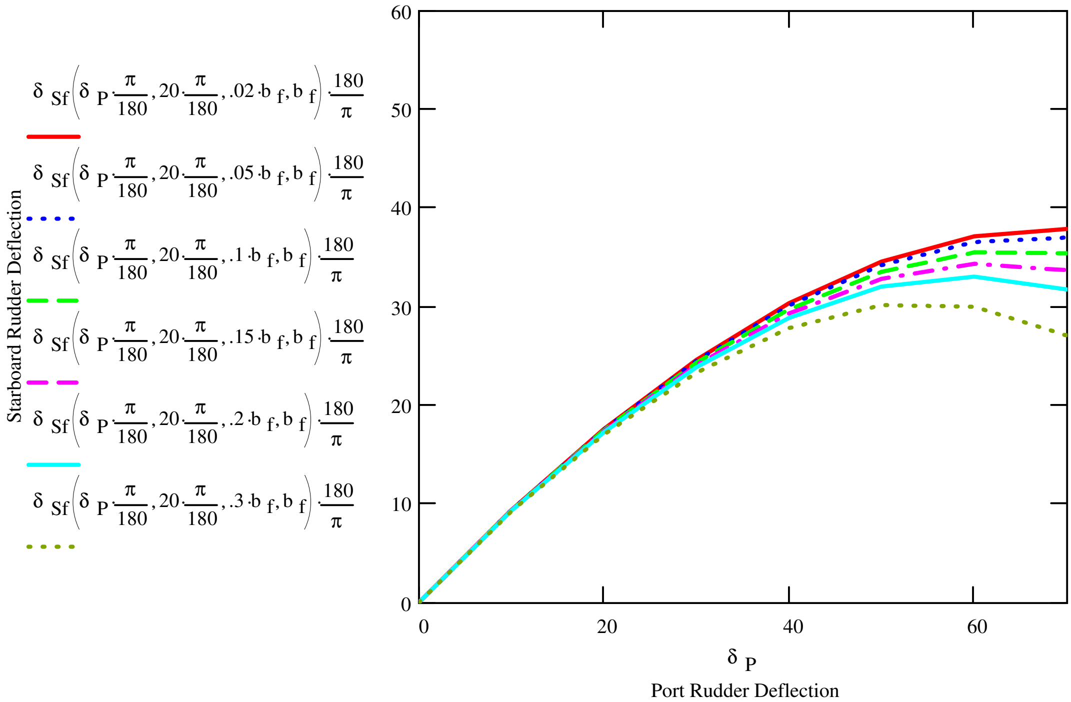
From Figure d, it's evident that there's a maximum deflection for the following rudder, and this is seen in the peaks of the curves. Where the curves bend back down, the linkage has become inverted. The longer the tillers compared to the beam, and the larger the Ackermann angle, the easier it is for the linkage to reach this limiting condition.

$b_f := 7.92 \cdot m \qquad r_{tf} := 1.89 \cdot m$

0 to 50 degrees Ackermann, Tillers 24% of beam



20 degree Ackermann, Tillers from 2% of beam to 30% of beam



Local Flow at Rudder Figure c.

Each rudder sees a different flow due to the motion of the boat. The inside rudder will be traveling slower than the outside rudder. This contribution is $\pm\frac{b}{2}\cdot\Omega$, with the sign depending on which rudder it is. Both rudders are being swept sideways by the turning of the boat, and this adds a velocity component $1\cdot\Omega$. The change in angle of attack due to this latter component always acts to oppose the turning of the boat. This is the yaw damping provided by the rudders.

The local angles of attack for the rudder are:

16. $$\alpha_P = \delta_P - \operatorname{atan}\left[\frac{V_b\cdot\sin(\lambda)+1\cdot\Omega}{V_b\cdot\cos(\lambda)-\frac{b}{2}\cdot\Omega}\right] = \delta_P - \operatorname{atan}\left[\frac{V_b\cdot\sin(\lambda)+1\cdot\frac{V_b}{R}}{V_b\cdot\cos(\lambda)-\frac{b}{2}\cdot\frac{V_b}{R}}\right]$$

17. $$\alpha_S = \delta_S - \operatorname{atan}\left[\frac{V_b\cdot\sin(\lambda)+1\cdot\Omega}{V_b\cdot\cos(\lambda)+\frac{b}{2}\cdot\Omega}\right] = \delta_S - \operatorname{atan}\left[\frac{V_b\cdot\sin(\lambda)+1\cdot\frac{V_b}{R}}{V_b\cdot\cos(\lambda)+\frac{b}{2}\cdot\frac{V_b}{R}}\right]$$

The Ackermann angle is to be designed so that the angles of attack on both rudders are the same. This ensures that they can both be operated on the edge of stall (α_{max}).

18. $$\delta_S - \operatorname{atan}\left[\frac{V_b\cdot\sin(\lambda)+1\cdot\frac{V_b}{R}}{V_b\cdot\cos(\lambda)+\frac{b}{2}\cdot\frac{V_b}{R}}\right] = \delta_P - \operatorname{atan}\left[\frac{V_b\cdot\sin(\lambda)+1\cdot\frac{V_b}{R}}{V_b\cdot\cos(\lambda)-\frac{b}{2}\cdot\frac{V_b}{R}}\right]$$

19. $$\delta_S = \delta_P - \operatorname{atan}\left[-2\cdot\frac{(\sin(\lambda)\cdot R+1)}{(-2\cdot\cos(\lambda)\cdot R+b)}\right] + \operatorname{atan}\left[2\cdot\frac{(\sin(\lambda)\cdot R+1)}{(2\cdot\cos(\lambda)\cdot R+b)}\right]$$

This is actually very close to the result obtained by Martin Schoon' s geometric analysis, but with the center of the turning circle shifted forward due to the leeway angle of the boat, and projected onto the boat' s centerline. This effectively lengthens the longitudinal distance to the rudders.

Since both rudders are to have the same angle of attack, the port rudder deflection can be determined from equation 17 and substituted into equation 19:

20. $$\delta_S = \alpha_P + \operatorname{atan}\left[2\cdot\frac{(\sin(\lambda)\cdot R+1)}{(2\cdot\cos(\lambda)\cdot R+b)}\right]$$

The following graphs show these relationships visually:

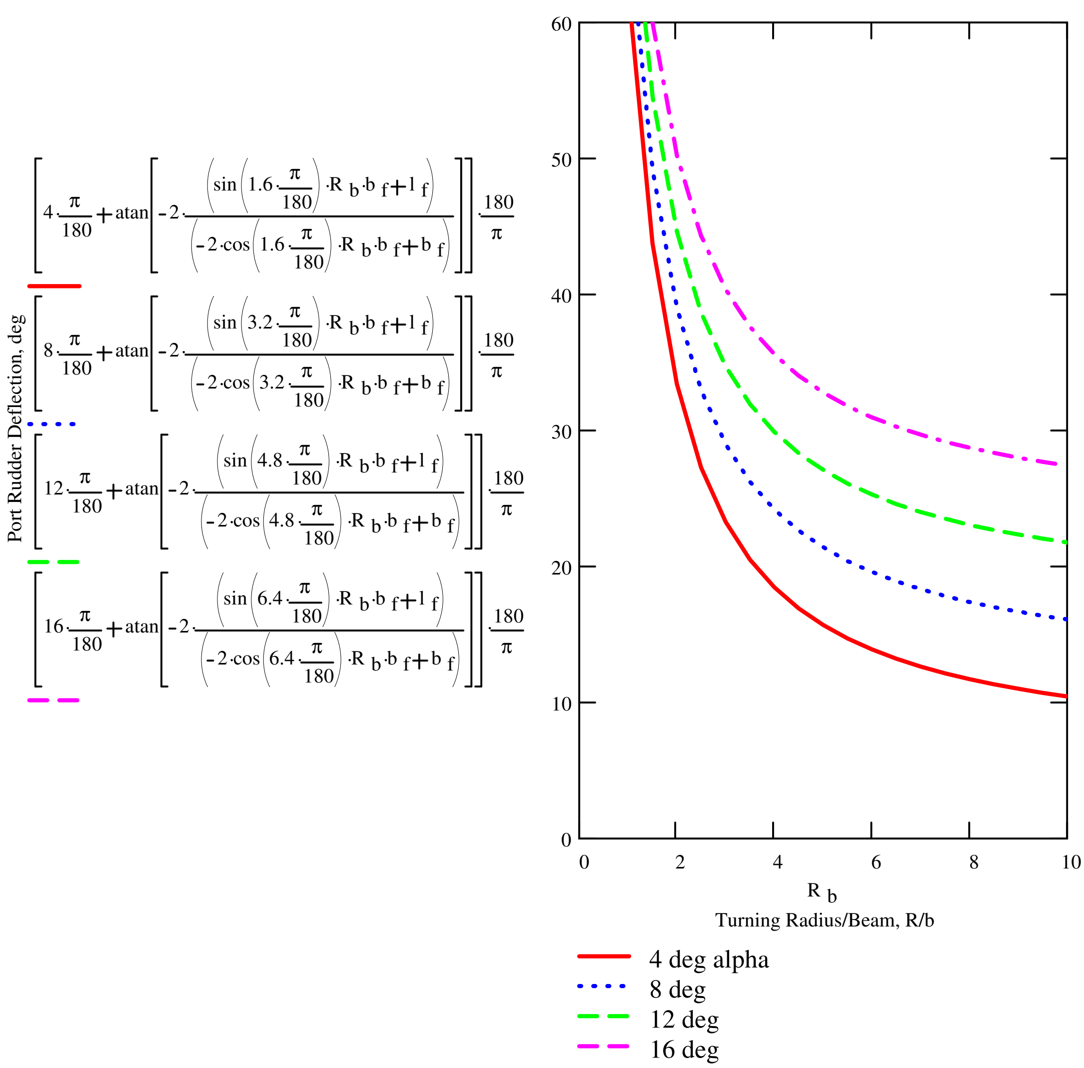
$$R_b := 1, 1.5 .. 10$$

$$\delta_{Sf2}(\alpha_P, \lambda, R, l, b) := \alpha_P + \operatorname{atan}\left[2 \cdot \frac{(\sin(\lambda) \cdot R + l)}{(2 \cdot \cos(\lambda) \cdot R + b)}\right]$$

$$\delta_{Sf3}(\alpha_P, \lambda, R, l, \gamma, r_t, b) := \delta_{Sf}\left[\alpha_P + \operatorname{atan}\left[-2 \cdot \frac{(\sin(\lambda) \cdot R + l)}{(-2 \cdot \cos(\lambda) \cdot R + b)}\right], \gamma, r_t, b\right]$$

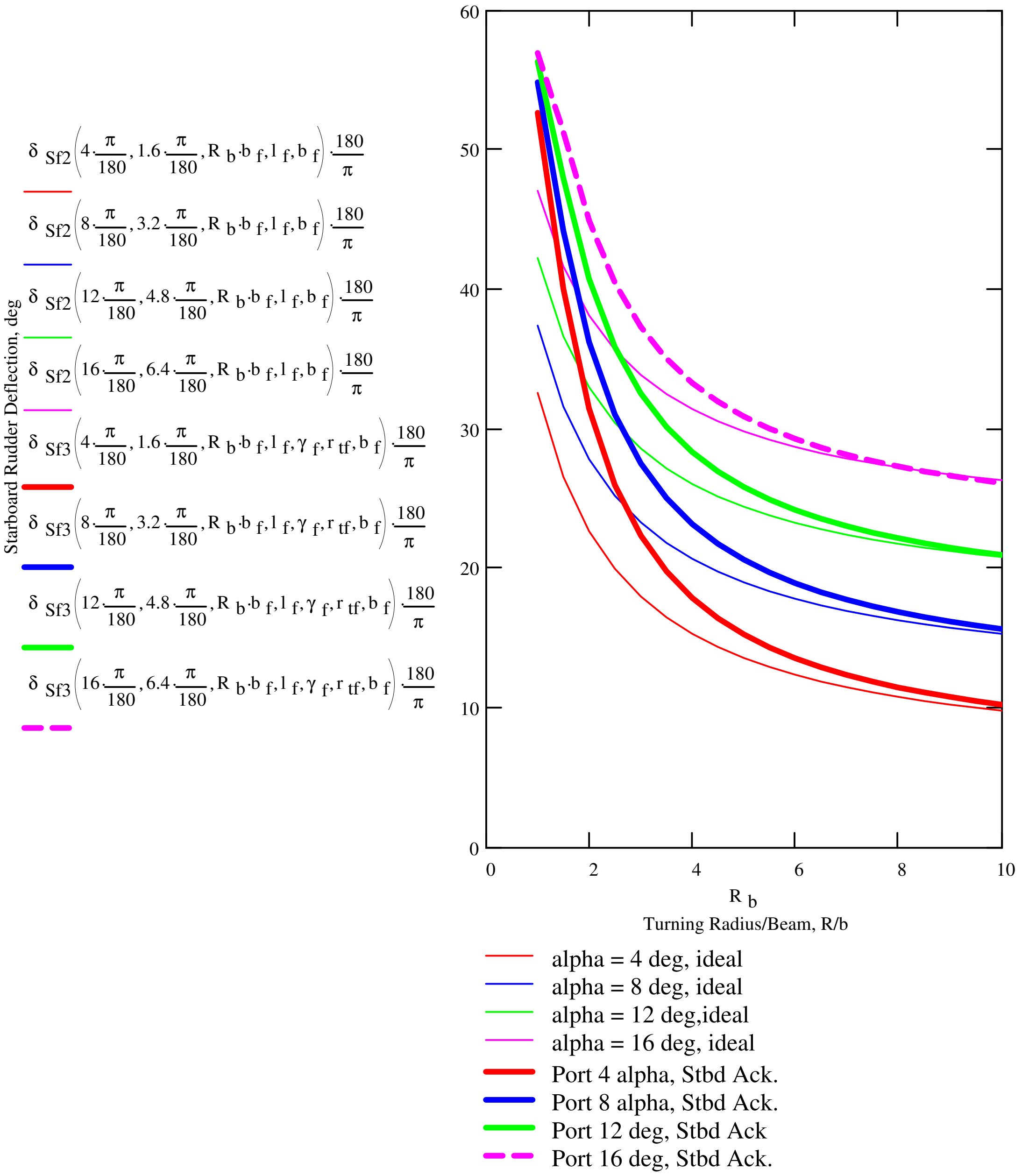
$$b_f := 7.92 \cdot m \qquad r_{tf} := 1.89 \cdot m \qquad l_f := 6.25 \cdot m$$

Port rudder deflection for various port rudder angles of attack



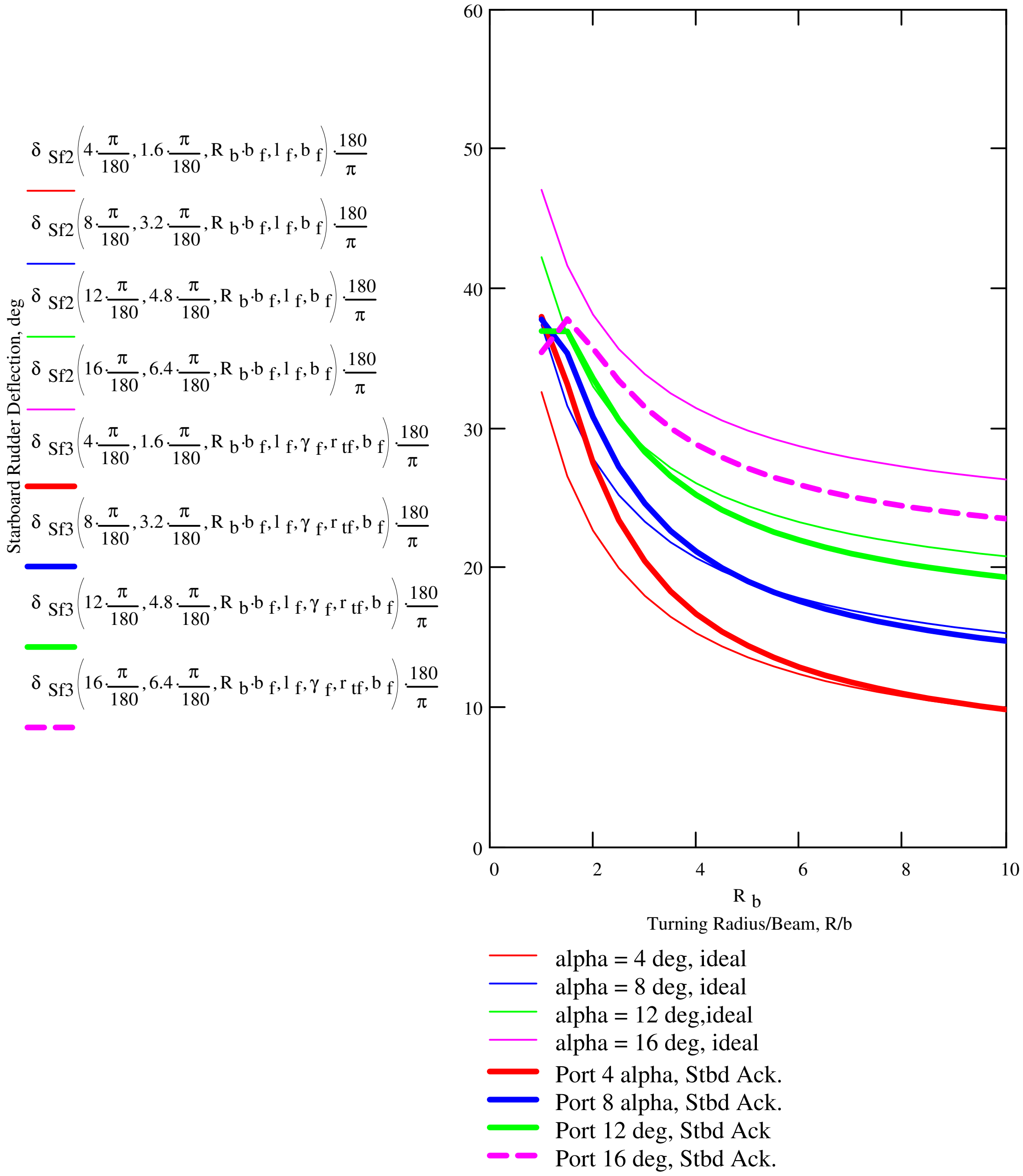
$$\gamma_f := 5 \cdot \frac{\pi}{180}$$

Starboard rudder deflection
5 degrees Ackermann angle
leeway 40% of rudder angle of attack
Thin lines = ideal angle
Heavy lines = Ackermann linked



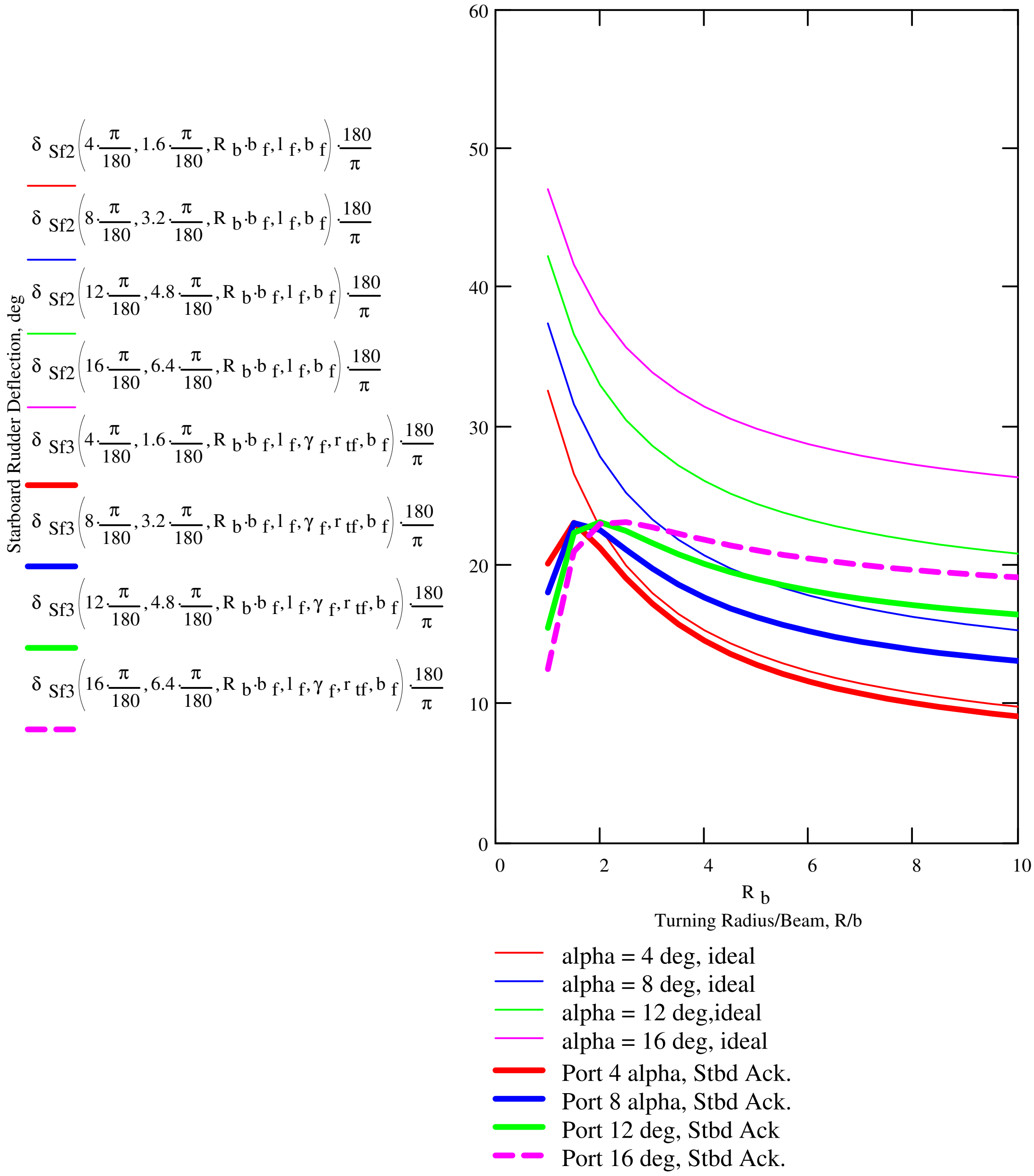
$$\gamma_f := 15 \cdot \frac{\pi}{180}$$

Starboard rudder deflection
15 degrees Ackermann angle
leeway 40% of rudder angle of attack
Thin lines = ideal angle
Heavy lines = Ackermann linked



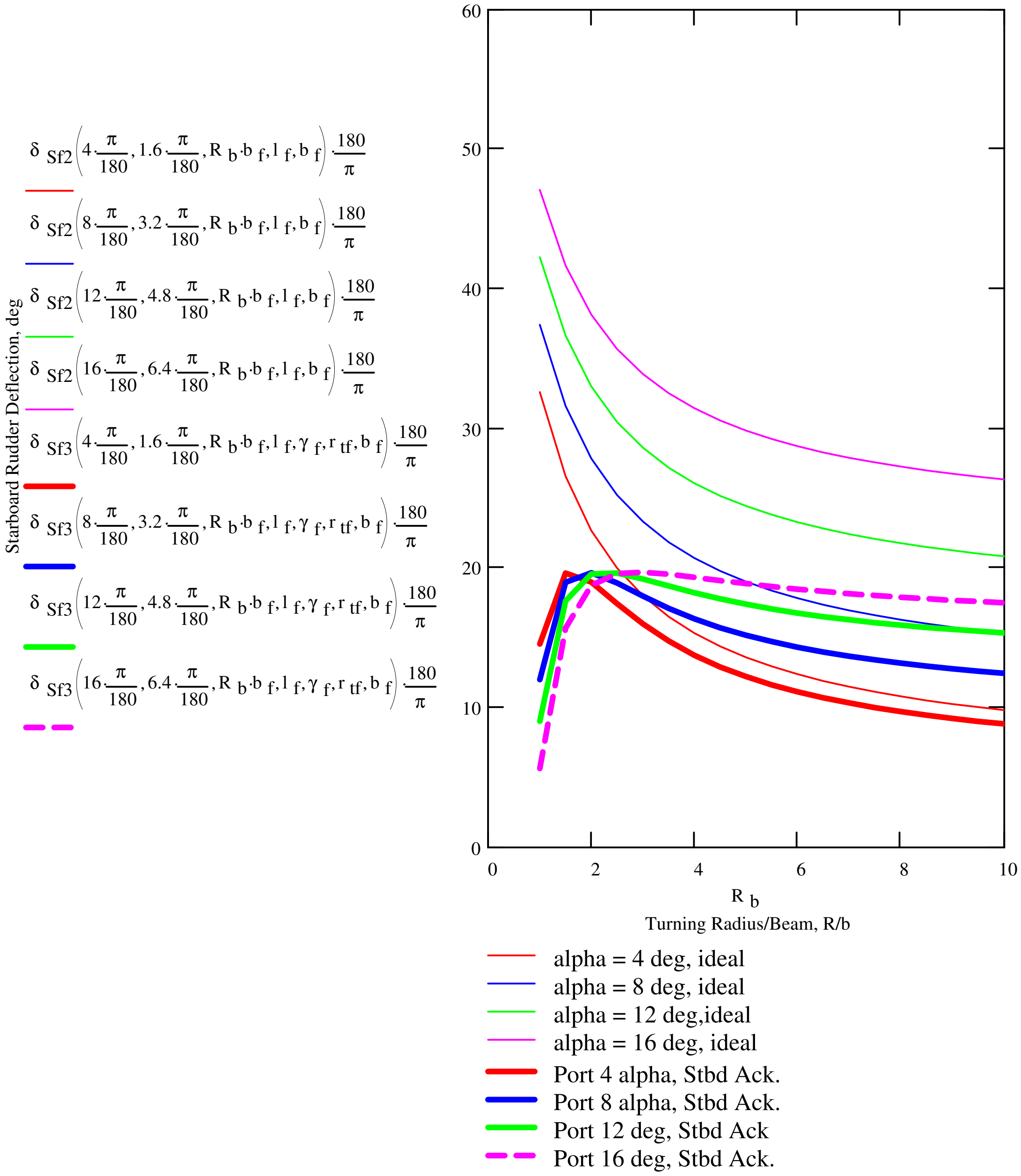
$$\gamma_f := 30 \cdot \frac{\pi}{180}$$

Starboard rudder deflection
30 degrees Ackermann angle
leeway 40% of rudder angle of attack
Thin lines = ideal angle
Heavy lines = Ackermann linked



$$\gamma_f := 35\cdot\frac{\pi}{180}$$

Starboard rudder deflection
35 degrees Ackermann angle
leeway 40% of rudder angle of attack
Thin lines = ideal angle
Heavy lines = Ackermann linked



Add Newton

So far, this analysis has shown how much the rudder deflection differs from port and starboard for different amounts of Ackermann angle and boat geometry. It' s also shown how much difference is needed based on the turning radius and the leeway angle made by the hull. However, one can get any amount of rudder deflection required depending on what one assumes for the turn radius. So the next step is determine the turning radius.

The centripetal acceleration of the boat is

21. $a = R \cdot \Omega^2 = V_b \cdot \Omega = \dfrac{V_b^{\,2}}{R}$ $\qquad\qquad g := 9.81 \cdot \dfrac{m}{s^2}$

And Newton' s law says that the sum of forces acting on the boat is equal to its mass times the acceleration (m is the mass, W the boat' s displacement, and g is the gravitational constant):

22. $L - L_P - L_S = a \cdot m = a \cdot \dfrac{W}{g}$

23. $L - L_P - L_S = \dfrac{V_b^{\,2}}{R} \cdot \dfrac{W}{g}$

The lift on the hull and keel comes from equation 5. The lift on the rudders is similar, but needs to account for the local speed and the fact that the local flow angles are not the same as the freestream. The local angles of attack for both rudders will be the same, by design. (S_r is the rudder area)

24. $L_P = C_{L\alpha} \cdot \alpha_P \cdot \dfrac{1}{2} \cdot \rho \cdot \left[V_b^{\,2} + (l \cdot \Omega)^2 - \left(\dfrac{b}{2} \cdot \Omega \right)^2 \right] \cdot S_r \cdot \cos\left[\operatorname{atan}\left[2 \cdot \dfrac{(\sin(\lambda) \cdot R + l)}{(2 \cdot \cos(\lambda) \cdot R - b)} \right] - \lambda \right]$

25. $L_S = C_{L\alpha} \cdot \alpha_S \cdot \dfrac{1}{2} \cdot \rho \cdot \left[V_b^{\,2} + (l \cdot \Omega)^2 + \left(\dfrac{b}{2} \cdot \Omega \right)^2 \right] \cdot S_r \cdot \cos\left[\operatorname{atan}\left[2 \cdot \dfrac{(\sin(\lambda) \cdot R + l)}{(2 \cdot \cos(\lambda) \cdot R + b)} \right] - \lambda \right]$

26. $L_P = C_{L\alpha} \cdot \alpha_P \cdot \dfrac{1}{2} \cdot \rho \cdot \left[V_b^{\,2} + \left(l \cdot \dfrac{V_b}{R} \right)^2 - \left(\dfrac{b}{2} \cdot \dfrac{V_b}{R} \right)^2 \right] \cdot S_r \cdot \cos\left[\operatorname{atan}\left[2 \cdot \dfrac{(\sin(\lambda) \cdot R + l)}{(2 \cdot \cos(\lambda) \cdot R - b)} \right] - \lambda \right]$

27. $L_S = C_{L\alpha} \cdot \alpha_S \cdot \dfrac{1}{2} \cdot \rho \cdot \left[V_b^{\,2} + \left(l \cdot \dfrac{V_b}{R} \right)^2 + \left(\dfrac{b}{2} \cdot \dfrac{V_b}{R} \right)^2 \right] \cdot S_r \cdot \cos\left[\operatorname{atan}\left[2 \cdot \dfrac{(\sin(\lambda) \cdot R + l)}{(2 \cdot \cos(\lambda) \cdot R + b)} \right] - \lambda \right]$

28. $L_P = C_{L\alpha} \cdot \alpha_P \cdot \dfrac{1}{2} \cdot \rho \cdot V_b^{\,2} \cdot S_r \cdot \left[1 + \dfrac{1}{R^2} \cdot l^2 - \dfrac{1}{\left(4 \cdot R^2\right)} \cdot b^2 \right] \cdot \cos\left[\operatorname{atan}\left[2 \cdot \dfrac{(\sin(\lambda) \cdot R + l)}{(2 \cdot \cos(\lambda) \cdot R - b)} \right] - \lambda \right]$

29. $L_S = C_{L\alpha} \cdot \alpha_S \cdot \dfrac{1}{2} \cdot \rho \cdot V_b^{\,2} \cdot S_r \cdot \left[1 + \dfrac{1}{R^2} \cdot l^2 + \dfrac{1}{\left(4 \cdot R^2\right)} \cdot b^2 \right] \cdot \cos\left[\operatorname{atan}\left[2 \cdot \dfrac{(\sin(\lambda) \cdot R + l)}{(2 \cdot \cos(\lambda) \cdot R + b)} \right] - \lambda \right]$

Putting all this together yields:

30.

$$C_{L\lambda}\cdot\lambda\cdot\frac{1}{2}\cdot\rho\cdot V_b^{\,2}\cdot S\ \ldots = \frac{V_b^{\,2}}{R}\cdot\frac{W}{g}$$

$$+ -C_{L\alpha}\cdot\alpha_P\cdot\frac{1}{2}\cdot\rho\cdot V_b^{\,2}\cdot S_r\cdot\left[1+\frac{1}{R^2}\cdot l^2-\frac{1}{\left(4\cdot R^2\right)}\cdot b^2\right]\cdot\cos\left[\operatorname{atan}\left[2\cdot\frac{(\sin(\lambda)\cdot R+l)}{(2\cdot\cos(\lambda)\cdot R-b)}\right]-\lambda\right]\ \ldots$$

$$+ -C_{L\alpha}\cdot\alpha_S\cdot\frac{1}{2}\cdot\rho\cdot V_b^{\,2}\cdot S_r\cdot\left[1+\frac{1}{R^2}\cdot l^2+\frac{1}{\left(4\cdot R^2\right)}\cdot b^2\right]\cdot\cos\left[\operatorname{atan}\left[2\cdot\frac{(\sin(\lambda)\cdot R+l)}{(2\cdot\cos(\lambda)\cdot R+b)}\right]-\lambda\right]$$

<- speed difference -><---- difference in flow angles ---->

31.

$$C_{L\lambda}\cdot\lambda_{max}\cdot\frac{1}{2}\cdot\rho\cdot S\ \ldots = 0$$

$$+ -C_{L\alpha}\cdot\alpha_{max}\cdot\frac{1}{2}\cdot\rho\cdot S_r\cdot\left[\left[1+\frac{1}{R^2}\cdot l^2-\frac{1}{\left(4\cdot R^2\right)}\cdot b^2\right]\cdot\cos\left[\operatorname{atan}\left[2\cdot\frac{\left(\sin\left(\lambda_{max}\right)\cdot R+l\right)}{\left(2\cdot\cos\left(\lambda_{max}\right)\cdot R-b\right)}\right]-\lambda_{max}\right]\ \ldots + \left[1+\frac{1}{R^2}\cdot l^2+\frac{1}{\left(4\cdot R^2\right)}\cdot b^2\right]\cdot\cos\left[\operatorname{atan}\left[2\cdot\frac{\left(\sin\left(\lambda_{max}\right)\cdot R+l\right)}{\left(2\cdot\cos\left(\lambda_{max}\right)\cdot R+b\right)}\right]-\lambda_{max}\right]\right]\ \ldots$$

$$+ -\frac{1}{R}\cdot\frac{W}{g}$$

Interestingly, the velocity cancels out and the minimum turning radius can be calculated based on the boat' s geometry and the stall angles for both the hull & keel and the rudders. There is only one speed at which the yawing moments will be balanced, however. But it isn' t necessary to know this to work out the Ackermann geometry.

As a first approximation to getting the turning radius, the fact that the lift on the rudders is not parallel to the lift on the hull can be neglected, and only the fact that the inside and outside rudders travel at different speeds is retained. With this simplification, equation 22 becomes:

32.

$$C_{L\lambda}\cdot\lambda_{max}\cdot\frac{1}{2}\cdot\rho\cdot S\ \ldots = 0$$

$$+ -C_{L\alpha}\cdot\alpha_{max}\cdot\frac{1}{2}\cdot\rho\cdot S_r\cdot\left[\left[1+\frac{1}{R^2}\cdot l^2-\frac{1}{\left(4\cdot R^2\right)}\cdot b^2\right]\ \ldots + \left[1+\frac{1}{R^2}\cdot l^2+\frac{1}{\left(4\cdot R^2\right)}\cdot b^2\right]\right]\ \ldots$$

$$+ -\frac{1}{R}\cdot\frac{W}{g}$$

32.

$$R_{min} = \frac{\left[W+\sqrt{W^2+2\cdot l^2\cdot\alpha_{max}\cdot C_{L\alpha}\cdot\rho^2\cdot S_r\cdot g^2\cdot\left(S\cdot\lambda_{max}\cdot C_{L\lambda}-2\cdot C_{L\alpha}\cdot\alpha_{max}\cdot S_r\right)}\right]}{\left[\rho\cdot\left[g\cdot\left(S\cdot\lambda_{max}\cdot C_{L\lambda}-2\cdot C_{L\alpha}\cdot\alpha_{max}\cdot S_r\right)\right]\right]}$$

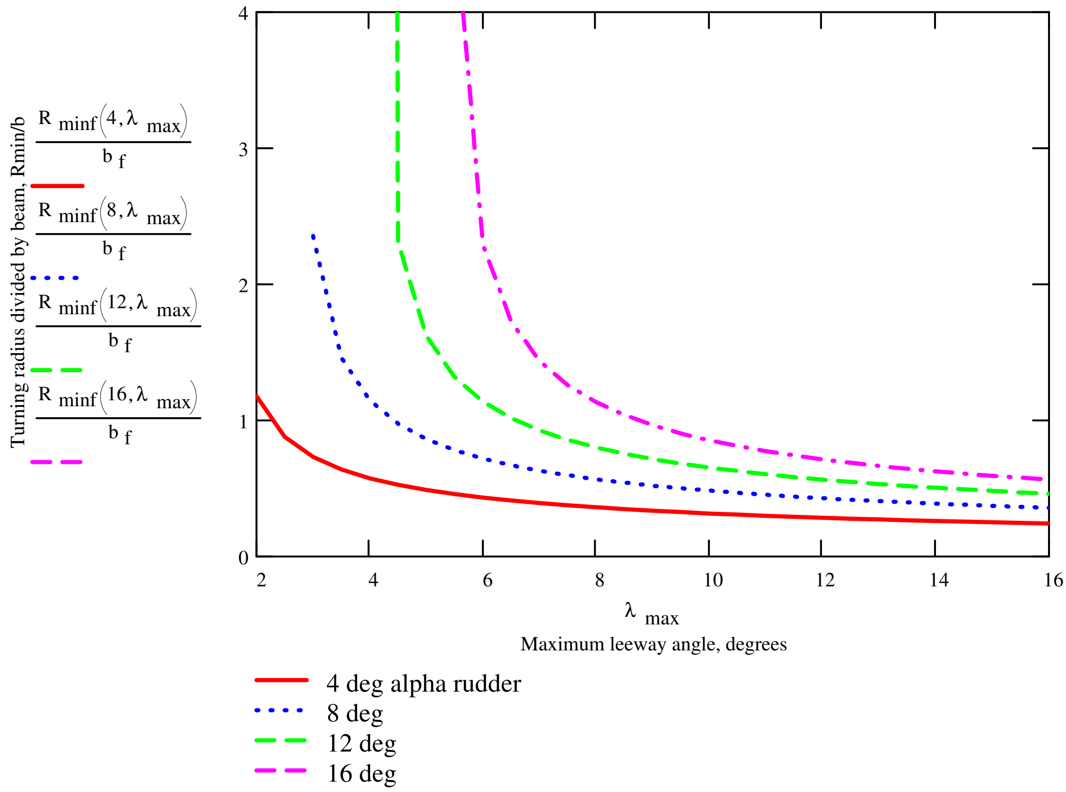
Substituting some typical numbers to see what all this means:

$W_f := 3700 \cdot kgf \quad b_f := 7.92 \cdot m \quad r_{tf} := 1.89 \cdot m \quad l_f := 6.25 \cdot m \quad \rho_f := 1000 \cdot kg \cdot m^{-3} \quad \lambda_{max} := 2, 2.5 .. 16$

$C_{L\alpha} := 0.07 \cdot deg^{-1} \quad C_{L\lambda} := 0.07 \cdot deg^{-1} \quad S_r := 0.495 \cdot m^2 \quad S_f := 2.97 \cdot m^2$

$$R_{minf}(\alpha_{max}, \lambda_{max}) := \frac{\left[W_f + \sqrt{W_f^2 + 2 \cdot l_f^2 \cdot \alpha_{max} \cdot C_{L\alpha} \cdot \rho_f^2 \cdot S_r \cdot g^2 \cdot (S_f \cdot \lambda_{max} \cdot C_{L\lambda} - 2 \cdot C_{L\alpha} \cdot \alpha_{max} \cdot S_r)} \right]}{\left[\rho_f \cdot \left[g \cdot (S_f \cdot \lambda_{max} \cdot C_{L\lambda} - 2 \cdot C_{L\alpha} \cdot \alpha_{max} \cdot S_r) \right] \right]}$$

Minimum turning radius in multiples of boat beam



The minimum turning radius goes down with increasing rudder angle of attack (deflection) because the force on the rudders is opposing the force on the keel. But some rudder is required to make the boat turn. This requires looking at the balance of the yawing moments. A boat with a lot of resistance to turning needs more angle of attack on the rudders and this saps its turning ability much like lee helm hurts windward ability.

But it appears that for the geometry assumptions made so far, for maximum leeway and rudder angles of attack on the order of 8 - 10 degrees the minimum turning radius would be on the order of one to two beam-widths and the most suitable Ackermann angle would be fairly small - on the order of 10 degrees to 15 degrees, depending on the maximum angle of attack. Too much Ackermann and the starboard rudder reverses at the high rudder deflections needed for tight turns.