# On the Tiller Geometry of Cats

Martin Schöön, October 1998 (in English Feb. 2001)

## Introduction

When turning with a catamaran the two hulls will move through the turn with different turning radii. The difference is equal to the centerline separation of the two hulls. The rudders of catamarans, with very few exemptions, are placed in the hulls or on the transoms of the hulls. It follows that the rudders should be angled differently. This paper deals with a geometrical way of deriving a fixed tiller linkage geometry to achieve good cooperation between the rudders of a cat. This geometry is often referred to as Ackermann geometry.

## Basic Geometry of a Turning Cat

Below I define the parameters I use for deriving the Ackermann geometry.

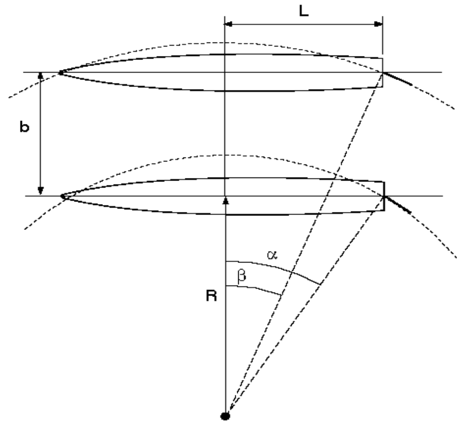



Fig. 1 Turning cat.

Key input parameters:

The hull centerline separation: $b := 4.65$

Distance between rudder and 'turning center': $L := 4.5$

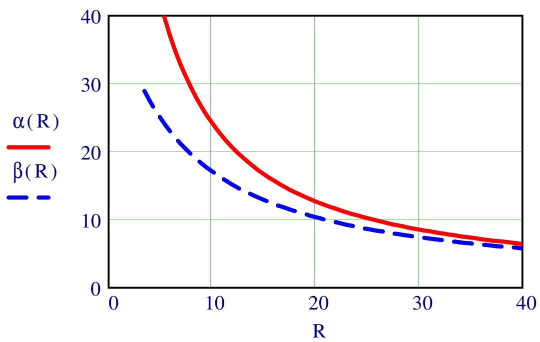
Turning radius for the inner hull: $R := 3.5, 4 .. 40$

Derived parameters:

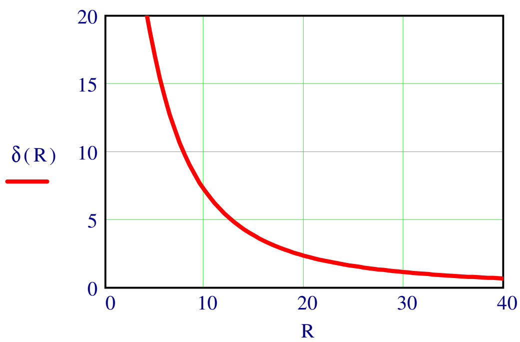
Turning radius of outer hull: $Ry(R) := R + b$

Inner and outer rudder angle: $\alpha(R) := \frac{180}{\pi} \cdot \operatorname{atan}\left(\frac{L}{R}\right)$ $\quad \beta(R) := \frac{180}{\pi} \cdot \operatorname{atan}\left(\frac{L}{Ry(R)}\right)$

Rudder angle difference: $\delta(R) := \alpha(R) - \beta(R)$ $\quad \alpha(41) = 6.263$ $\quad \beta(41) = 5.63$




Plot 1. Rudder angles




Plot 2. Difference between rudders

Note that the difference between the rudders is rather big for small turning radii - when tacking or maneuvering in tight spots. For course adjustments on open water the difference is really small. For a turning radius of 100 m it is in the order of 1/10 of a degree: $\delta(100) = 0.114$

## Tiller Geometry and Solving for 'Optimal' Ackermann

Now let us take a closer look at the geometry of the tillers and their connection bar.

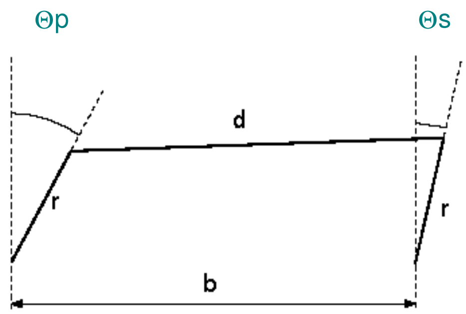



Fig. 2 Tiller geometry.

Length of tiller: $r := 1.0$

Ackermann: $\gamma := 0, 5 .. 40$

Tiller connection bar length: $d(\gamma) := b - 2 \cdot r \cdot \sin\left(\pi \cdot \frac{\gamma}{180}\right)$

In Fig. 2 Θp is the angle of the port tiller and it is equal to port rudder angle plus Ackermann (Θp = α + γ). In the same manner the angle of the starboard tiller is Θs = β - γ.

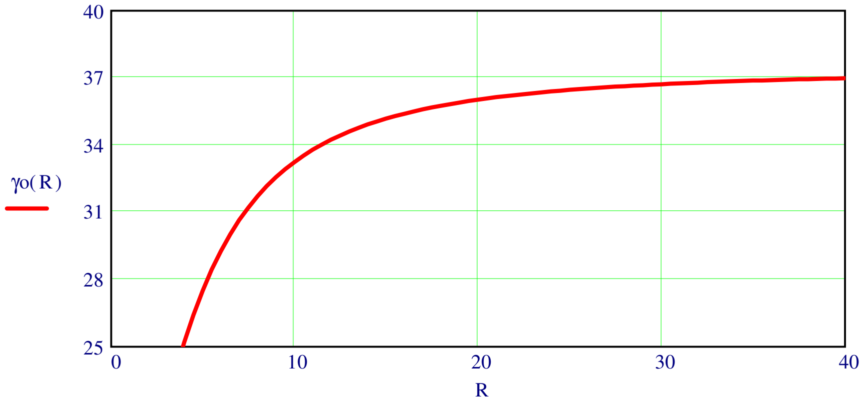
We now have two ways to calculate the length of the of the tiller connection bar. The first one is the one above based on the ' going straight' tiller geometry. The second one can be based on the relation between turning radii and rudder angle and, tiller angles that are functions of these factors and the Ackermann angle. The correct Ackermann angle for a given turning radius and boat geometry is the one that gives the same connection bar length in both calculations. Put another way: We subtract one connection bar length equation from the other and look up the Ackermann angle that makes the subtraction result in a zero. I cannot solve this with ' pen and paper' so I make Mathcad solve this numerically:

$\gamma f := 30$ (' seed' )

$$\text{funk}(\gamma f) := \begin{array}{|l} \Theta s \leftarrow (\beta(R) - \gamma f)\cdot\frac{\pi}{180} \\ \Theta p \leftarrow (\alpha(R) + \gamma f)\cdot\frac{\pi}{180} \\ \sqrt{[b - r\cdot(\sin(\Theta p) - \sin(\Theta s))]^2 + r^2\cdot(\cos(\Theta p) - \cos(\Theta s))^2} - b + 2\cdot r\cdot\sin\left(\frac{\pi}{180}\cdot\gamma f\right) \end{array}$$

The last row above is the subtraction of the two connection bar equations. Now I ask Mathcad to look for Ackermann angles that result in a zero:

$\gamma o(R) := \text{root}(\text{funk}(\gamma f), \gamma f)$




Plot 3. Optimal Ackermann

As we see the optimal Ackermann angle is depending to a rather large degree on how sharp one turns and the optimum angle increases with increasing radius. This might come as a surprise (did to me) but all is in order. A more useful and hopefully less puzzling way to study this is to plot the errors that fixed geometries result in. This is after all what we would have to live with on the boat.

## Resulting Error with Fixed Ackermann

In the following I have assumed that the outer rudder tracks correctly and I calculate the error of the inner rudder as a function of turning radius and Ackermann angle. (Well, this is just the result. The actual derivation was done on paper some 15 years ago.)

Starboard (outer) tiller angle: $\Theta s(R, \gamma) := (\beta(R) - \gamma)\cdot\frac{\pi}{180}$

$$N(R,\gamma) := 4\cdot b\cdot r\cdot\sin(\Theta s(R,\gamma)) - 2\cdot d(\gamma)^2 + 2\cdot b^2 + 4\cdot r^2 + 4\cdot r^2\cdot\cos(\Theta s(R,\gamma))$$

$$A(R,\gamma) := 4\cdot b\cdot r + 4\cdot r^2\cdot\sin(\Theta s(R,\gamma))$$

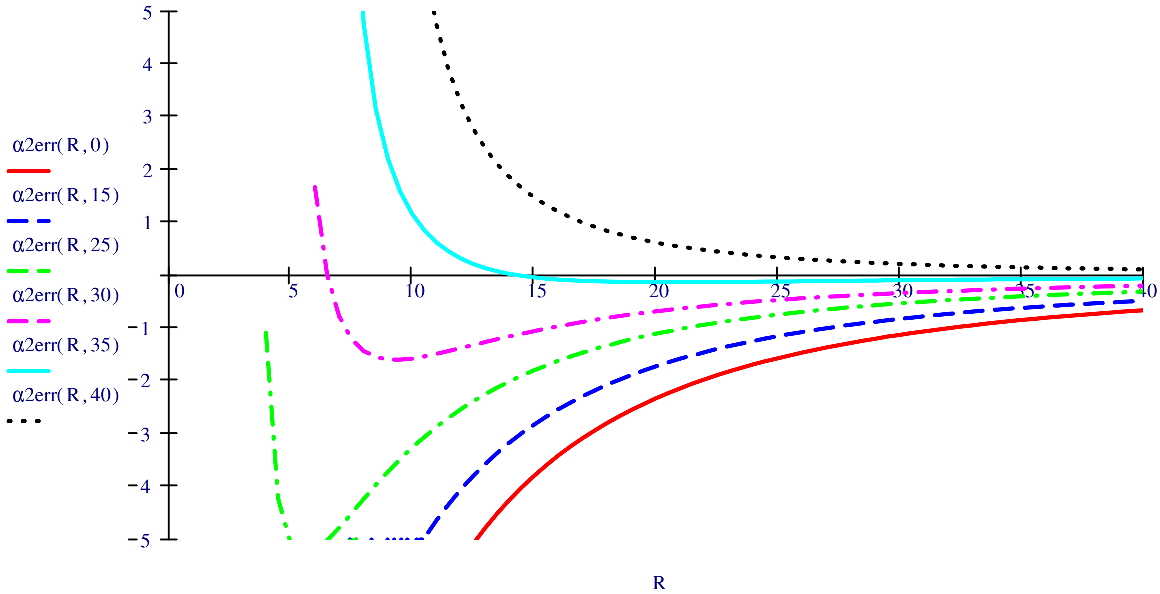
$$B(R,\gamma) := \left(\sqrt{-4\cdot b^3\cdot r\cdot\sin(\Theta s(R,\gamma)) + 4\cdot b\cdot r\cdot\sin(\Theta s(R,\gamma))\cdot d(\gamma)^2 + 4\cdot r^2\cdot\cos(\Theta s(R,\gamma))^2\cdot b^2 - 4\cdot b^2\cdot r^2 + 4\cdot d(\gamma)^2\cdot r^2 + 2\cdot d(\gamma)^2\cdot b^2 - d(\gamma)^4 - b^4}\right)\cdot 2$$

Two port tiller solutions are mathematically possible:

$$\Theta p1(R,\gamma) := 2 \cdot \operatorname{atan}\left[\frac{1}{N(R,\gamma)} \cdot \left(A(R,\gamma) + B(R,\gamma)\right)\right] \qquad \Theta p2(R,\gamma) := 2 \cdot \operatorname{atan}\left[\frac{1}{N(R,\gamma)} \cdot \left(A(R,\gamma) - B(R,\gamma)\right)\right]$$

Resulting in two possible errors for the port rudder angle:

$$\alpha 1err(R,\gamma) := \Theta p1(R,\gamma) \cdot \frac{180}{\pi} - \gamma - \alpha(R) \qquad \alpha 2err(R,\gamma) := \Theta p2(R,\gamma) \cdot \frac{180}{\pi} - \gamma - \alpha(R)$$



Plot 4. Inner rudder angle errors

Positive error means the inner rudder oversteers and vice versa. In Plot 4 we see that the optimum Ackermann in Plot 3 is the one that makes the curve cross zero at a certain turning radius (the 35 degree curve cross zero at ~10m which checks with the plot of optimum angles). We also see that for this boat an Ackermann angle close to 35 degrees is a good choice and that someone who has tried much smaller Ackermann angles (15 degrees or less) is quite likely to conclude that it wasn' t worth the effort.

## Disclaimer and Proposal for Further Investigations

This analysis is purely geometrical and does not take actual forces, speeds and accelerations into account. Ultimately all this should be included in the analysis and the tiller geometry that results in the smallest resistance for a given turning radius should be sought.