Some aspects of structural engineering of dagger boards.

Martin Schöön, Dec. 2004

Foreword

This is not ment to be an exhaustive treatment of the structural engineering of dagger boards. My intention is only to give you some insight into how complex even the structural engineering of dagger boards is. I have only dealt with two load cases and I don't take the shear stiffness of the web into consideration. A carbon/epoxy design is studied since these materials are not prohibitively expensive any more.

Disclaimer

Material data used here should be regarded as examples.

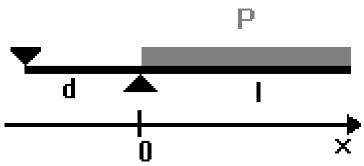
First load case: Hard on the wind, dagger boards fully down

$RM := 30 \cdot 10^{3} \cdot newton \cdot m$ Righting moment from stability calc.

$CSA := 5.5 \cdot m$ Height of centre of effort of sail plan.

$FRM := \frac{RM}{CSA}$ Side force countered by board. I think it is reasonable to have one board take this load alone. Wave action is the reason.

Geometry and Load Input



Length in water: $l := 1.4 \cdot m$

Length in box: $d := 1.0 \cdot m$

Thickness: $H := 60 \cdot mm$

Cord: $c := 0.45 \cdot m$

Area: $A := l \cdot c$ $A = 0.63 \cdot m^{2}$

Load from stab.: $PRM := \frac{FRM}{A}$ $PRM = 8.658 \cdot 10^{3} \cdot Pa$

Water density: $\rho := 1 \cdot \frac{kg}{liter}$

Material data, unidirectional carbon in epoxy

Young's modulus: $E := 62 \cdot 10^{9} \cdot Pa$

Ultimate stress: $Rp := 620 \cdot 10^{6} \cdot Pa$

Strain at fatigue stress: $su := 0.3 \cdot \%$

Fatigue stress: $Ru := E \cdot su$

$Ru = 1.86 \cdot 10^{8} \cdot Pa$

This data is taken from "Marine Composites" by Eric Greene Associates and I use data for compression loads since this is the worst data.
The material builds 0.53 mm for each 350 g layer.
The ultimate strain for carbon fibres is some 1% and for
good epoxy I find values between 3% and 6%. I have used 10x safety margin on 3% to calculate an acceptable stress level (fatigue stress). This is not very scientific!!!

Here I just create a convenient variable x that I sweep along the board length.

$J := 241$ $j := 0 .. J$ $d + l = 2.4 \cdot m$ $x_{j} := -d + \frac{d + l}{J} \cdot j$

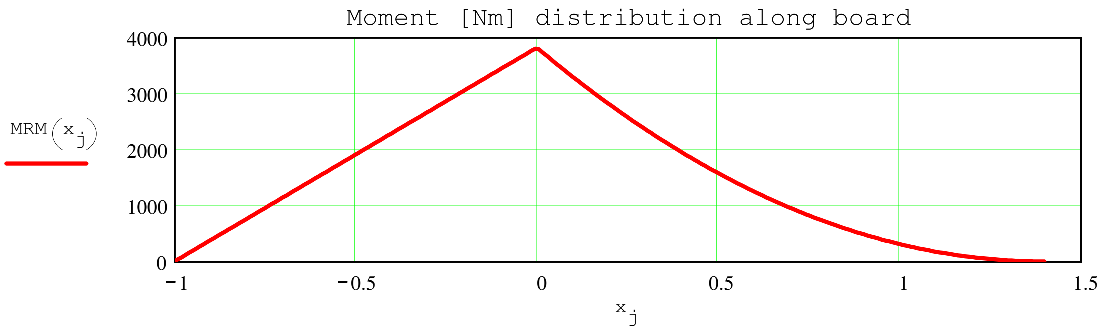
Structural 'textbook' equations

Moment:
$$MRM(x) := \begin{cases} \frac{c \cdot PRM \cdot (l - x)^{2}}{2} & \text{if } x > 0 \\ \frac{c \cdot PRM \cdot l^{2}}{2} \cdot \frac{d + x}{d} & \text{otherwise} \end{cases}$$

Outside box

$MRM(0 \cdot m) = 3.818 \cdot 10^{3} \cdot m \cdot newton$

Inside box



Loads on the dagger board case

Total side-load from righting moment: $FRM = 5.455 \cdot 10^{3} \cdot newton$

Load at upper end of board: $Fu := \frac{MRM(0 \cdot m)}{d}$ $Fu = 3.818 \cdot 10^{3} \cdot newton$

Load at hull bottom: $Fb := FRM + Fu$ $Fb = 9.273 \cdot 10^{3} \cdot newton$

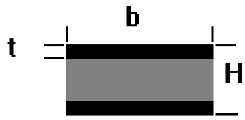
Note The load at the lower end of the dagger board case is equivalent to having the hull laying down on its side and putting a 944 kg weight at that case exit. Make sure the boat is stronger than the boards!

Engineering of board structural member

Beam width: $b := 10 \cdot cm$ Laminate thickness: $t := 4.24 \cdot mm$ (guess)

Sectional stiffness: $I := \frac{b \cdot \left[H^{3} - (H - 2 \cdot t)^{3}\right]}{12}$

Sectional strength: $w := \frac{I \cdot 2}{H}$



Max stress in design above: $\sigma max := \frac{MRM(0 \cdot m)}{w}$ $\sigma max = 1.734 \cdot 10^{8} \cdot Pa$

Calculating required minimum laminate thickness

by going from moment and strength via section properties:

From RM: $wmin(x) := \frac{MRM(x)}{Ru}$ $Imin(x) := H \cdot \frac{wmin(x)}{2}$

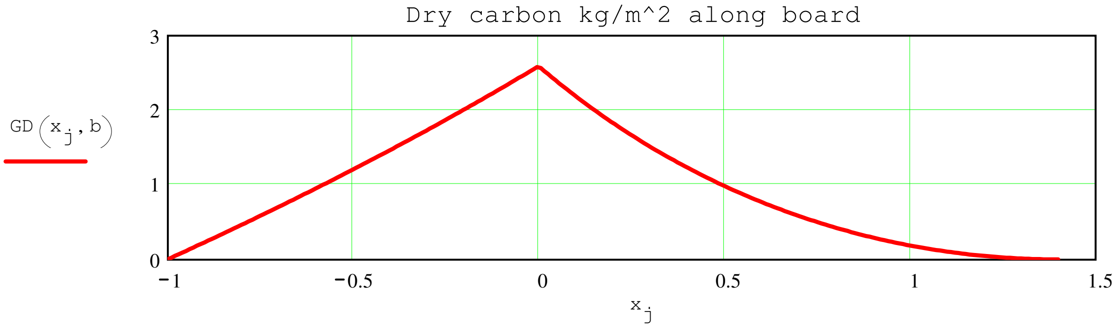
$$tmin(x, b) := 0.5 \cdot \left[H - \left(H^{3} - \frac{12 \cdot Imin(x)}{b}\right)^{\frac{1}{3}}\right]$$

$tmin(0 \cdot m, b) = 3.908 \cdot 10^{-3} \cdot m$

Fibreweight/m² calculation

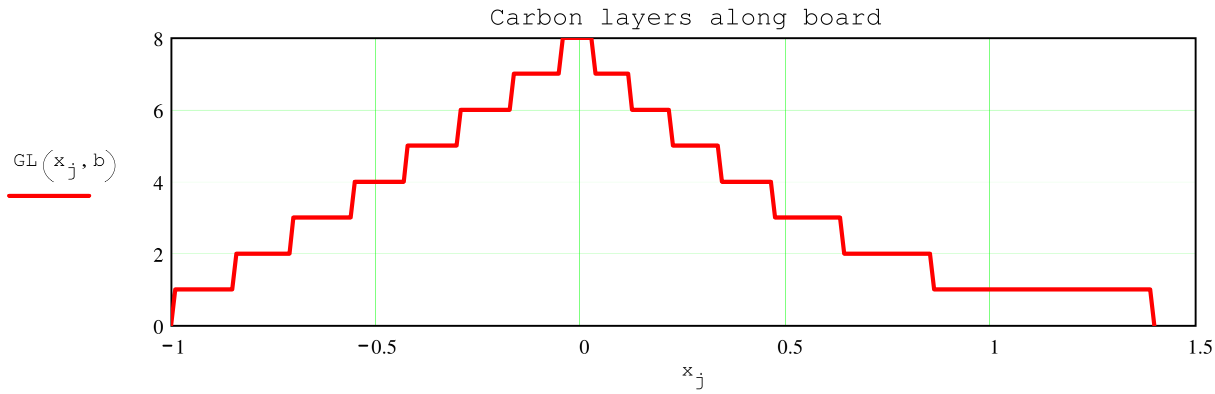
Assumed thickness ' density' :

$$TGD := \frac{1.51 \cdot 10^{-3} \cdot m}{1 \cdot \frac{kg}{m^2}} \qquad GD(x,b) := \frac{tmin(x,b)}{TGD}$$



Calculating how many layers that are required

$$fibreweight := 0.35 \cdot kg \cdot m^{-2} \qquad GL(x,b) := ceil\left[\frac{Re((GD(x,b)))}{fibreweight}\right]$$



Shear web compression at hull exit

Whatever keeps the two sides of the board apart must be hard enough not to crush at the hull exit.

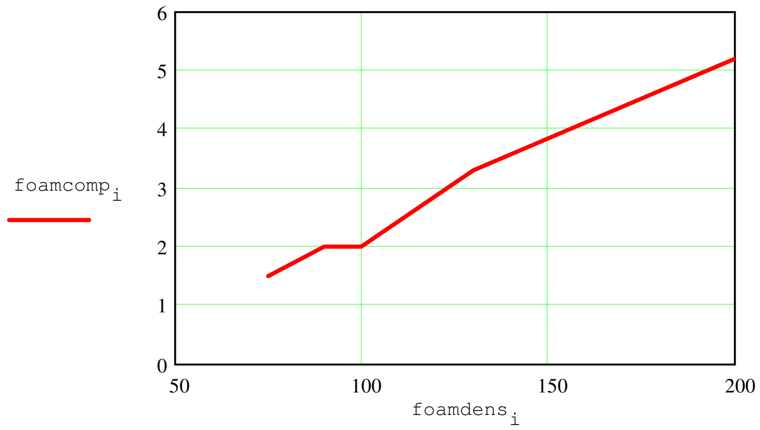
Pressure assuming some 5 cm of the beam length is really involved:

$$Pweb(b) := \frac{Fb}{b \cdot 50 \cdot mm} \qquad Pweb(b) = 1.855 \cdot 10^{6} \circ Pa$$

$i := 0 .. 4$

$foamdens := (75 \quad 90 \quad 100 \quad 130 \quad 200)^{T}$ kg/m^3 Termanto (PVC foam)

$foamcomp := (1.5 \quad 2 \quad 2 \quad 3.3 \quad 5.2)^{T}$ MPa



Looks like the 100 kg foam is the lightest we can use. Most woods including end grain balsa would be more than strong enough but must be carefully protected from water. Balsa also has a much higher shear modulus, something that has a considerable impact on the stiffness of the board. NB With 100 kg foam there is not much margin for shock loads etc. I use 130 kg foam in the rest of this document.

Board deflection while going to windward

I now look at the board deflection and take into account the fact that its stiffness is a function of the co-ordinate along the board. I start by calculating the moment of inertia from the laminate as built.

$$GD\_build(x,b) := GL(x,b)\cdot fibreweight$$

$$t\_build(x,b) := GD\_build(x,b)\cdot TGD$$

$$I\_build(x,b) := \frac{b\cdot\left[H^3 - (H - 2\cdot t\_build(x,b))^3\right]}{12}$$

Text book equation for the deflection of a homogenous console beam with uniform load:

$$\frac{FRM\cdot(l-x)}{24\cdot EIl}\cdot\left[4\cdot l^3 - (l-x)^3\right]$$

Now this equation has been differentiated with respect to x and I_build has been introduced:

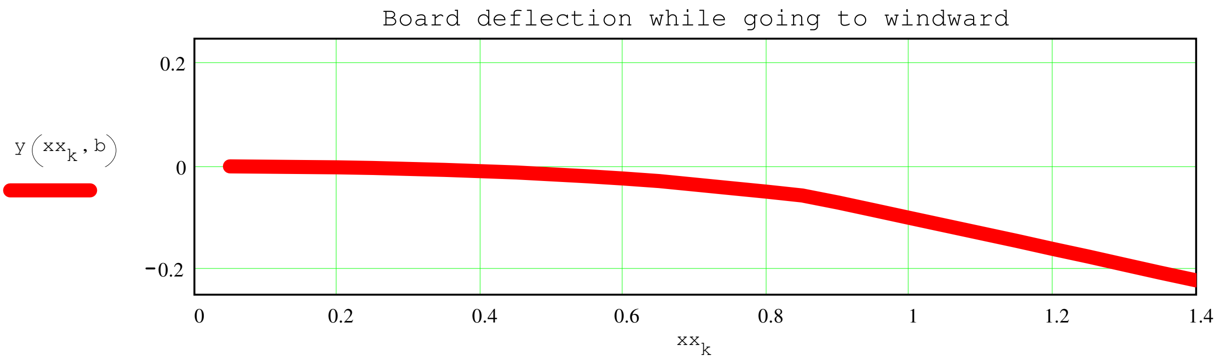
$$yd(x,b) := \frac{-1}{24}\cdot\frac{FRM}{E\cdot I\_build(x,b)\cdot l}\cdot\left[4\cdot l^3 - (l-x)^3\right] + \frac{1}{8}\cdot FRM\cdot\frac{(l-x)^3}{E\cdot I\_build(x,b)\cdot l}$$

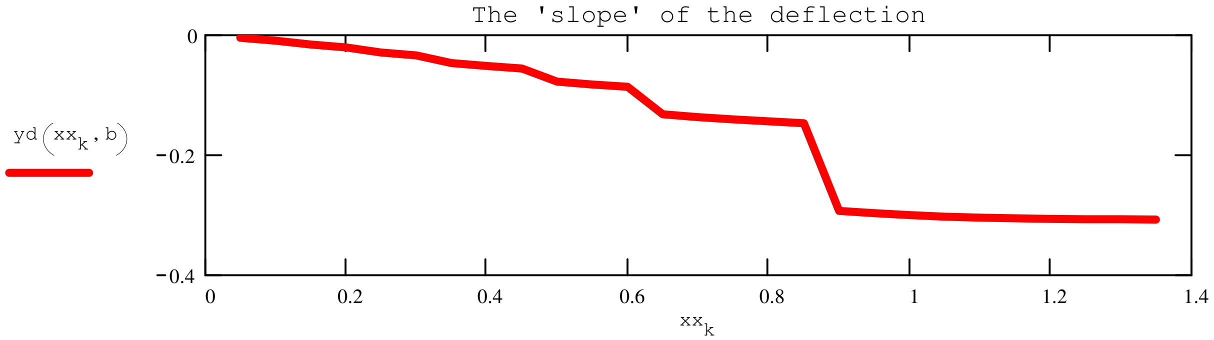
Deflection of the board as a function of x now becomes:

$$y(x,b) := \int_0^x \frac{-1}{24}\cdot\frac{FRM}{E\cdot I\_build(x,b)\cdot l}\cdot\left[4\cdot l^3 - (l-x)^3\right] + \frac{1}{8}\cdot FRM\cdot\frac{(l-x)^3}{E\cdot I\_build(x,b)\cdot l}\,dx$$

Board tip deflection: $y(l - 0.001\cdot m, b) = -0.222\cdot m$

$k := 1..28 \qquad xx_k := 0 + k\cdot 0.05\cdot m$





The nonconstant stiffness made this a bit complex and I have yet to take into account the finite shear modulous of the shear web.

Note how uneven the ' slope' curve is. Each step corresponds to a local stress peak. It really pays off to taper the ends of each laminate layer to minimise stress concentration.

The shear stress in the foam while going to windward

Here I calculate the shear stress for the foam in the sandwich. NB This calculation is really only true for a homogenous beam. What happens for tapered beams I must find out.

Shear modulus, shear strength and Young' s modulous of 130 kg foam:

$$G := 41 \cdot 10^{6} \cdot Pa \qquad \tau := 2.4 \cdot 10^{6} \cdot Pa$$

$$Ef := 0.19 \cdot 10^{9} \cdot Pa$$

$$\tau f(b) := \frac{FRM}{H \cdot b} \qquad \tau f(b) = 9.091 \cdot 10^{5} \cdot Pa \qquad \frac{\tau f(b)}{\tau} = 37.879 \cdot \%$$

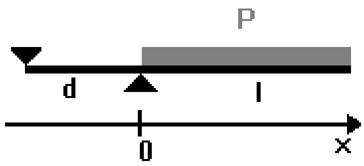
% of ultimate shear strength. Is this enough? I doubt it.

Second load case: broad reaching with boards halfway down

In this scenario we broad reach at ' full speed' with both boards halfway down. Steep waves (wind against tide?) make sure the boards see high and opposing loads.

Length in water: $l := \frac{l}{2}$

Length in box: $d := 1.0 \cdot m$



Area: $A := l \cdot c$

$A = 0.315 \cdot m^2$

$Vmax := 10 \cdot \frac{m}{sec}$ $\quad Clmax := 1.2$ $\quad Fmax := 0.5 \cdot A \cdot Clmax \cdot \rho \cdot Vmax^2$ $\quad Pmax := \frac{Fmax}{A}$

New swept x:

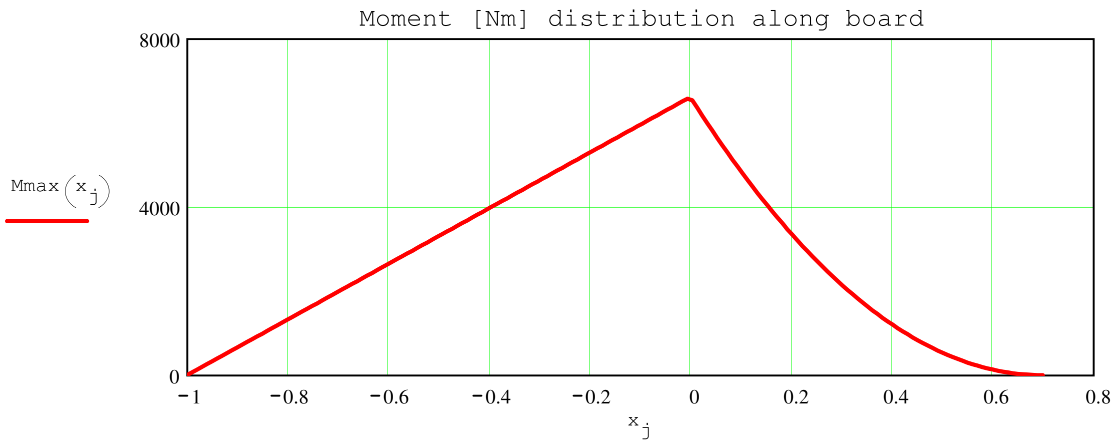
$J := 171$ $\quad j := 0 .. J$ $\quad d + l = 1.7 \cdot m$ $\quad x_j := -d + \frac{d + l}{J} \cdot j$

$$Mmax(x) := \begin{cases} \dfrac{c \cdot Pmax \cdot (l - x)^2}{2} & \text{if } x > 0 \\ \dfrac{c \cdot Pmax \cdot l^2}{2} \cdot \dfrac{d + x}{d} & \text{otherwise} \end{cases}$$

Outside box

$Mmax(0 \cdot m) = 6.615 \cdot 10^3 \cdot m \cdot newton$

Inside box



Total side-load: $Fmax = 1.89 \cdot 10^4 \cdot newton$

Load at upper end of board: $Ful := \frac{Mmax(0 \cdot m)}{d}$ $\quad Ful = 6.615 \cdot 10^3 \cdot newton$

Load at hull bottom: $Fbl := Fmax + Ful$ $\quad Fbl = 2.552 \cdot 10^4 \cdot newton$

$wmin\_h(x) := \frac{Mmax(x)}{Ru}$ $\quad Imin\_h(x) := H \cdot \frac{wmin\_h(x)}{2}$

$$tmin\_h(x) := 0.5 \cdot \left[H - \left(H^3 - \frac{12 \cdot Imin\_h(x)}{b} \right)^{\frac{1}{3}} \right]$$

$tmin\_h(0 \cdot m) = 7.763 \cdot 10^{-3} \cdot m$

It is clear from this that one should not subject the boat to this kind of load and count on it taking it forever unless it is very robust. Hence, at least one of the boards should be raised almost completely if you sail for any length of time at high speed in rough conditions!

Another conclusion one could draw would be to taper the laminate less aggressively towards the bottom end. A side benefit from this is less deflection. For my earlier designs, that were not designed using this kind of ' number crunching' , I have used linear tapers for the laminate. This adds almost nothing to the laminate close to the max but it beefs up the lower parts of the board.

Analysis of altered design

A linear laminate taper is used for better ' tip robustness' and less flex. I also widen the structural member to lower the stress on the foam and the dagger board case.

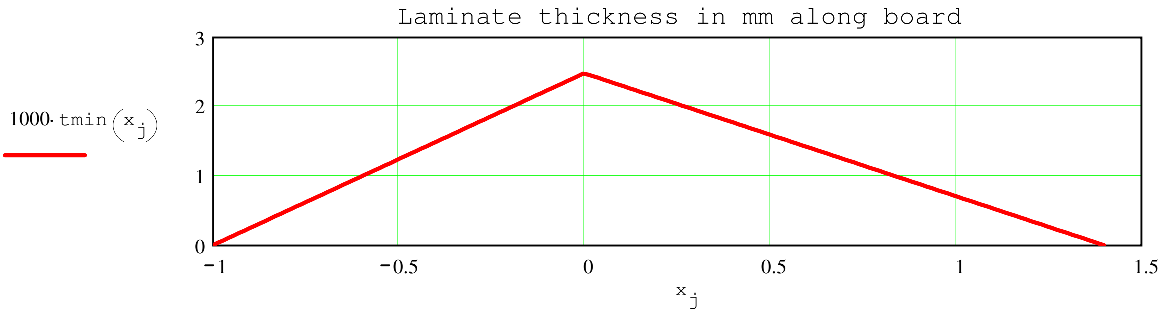
$$Ru := 186 \cdot 10^6 \cdot Pa \qquad b := 0.15 \cdot m \qquad l := 1.4 \cdot m \qquad d := 1.0 \cdot m$$

$$j := 0 .. J \qquad d + l = 2.4 \cdot m \qquad x_j := -d + \frac{d + l}{J} \cdot j$$

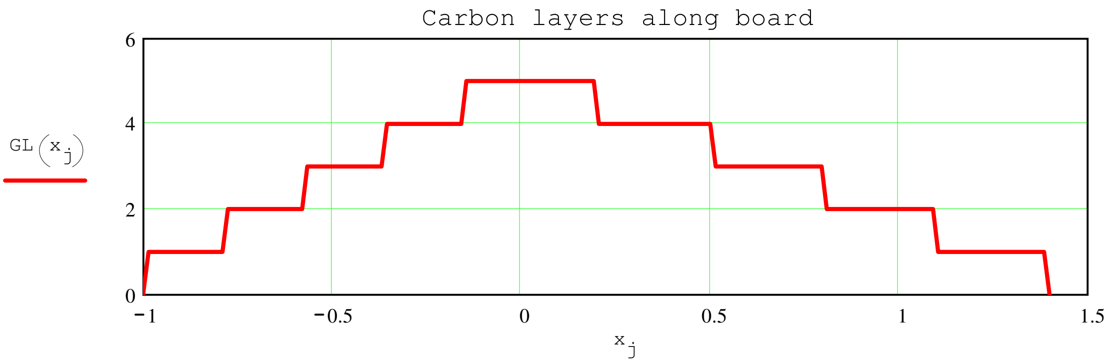
$$J := 241$$

$$wmin := \frac{MRM(0 \cdot m)}{Ru} \qquad Imin := H \cdot \frac{wmin}{2} \qquad tmin := 0.5 \cdot \left[H - \left(H^3 - \frac{12 \cdot Imin}{b} \right)^{\frac{1}{3}} \right] \qquad 100 \cdot \frac{Ru}{E} = 0.3 \, \%$$

$$tmin(x) := \begin{cases} tmin + \dfrac{tmin \cdot x}{d} & \text{if } x < 0 \\ tmin - \dfrac{tmin \cdot x}{l} & \text{otherwise} \end{cases}$$



$$GD(x) := \frac{tmin(x)}{TGD} \qquad GL(x) := \text{ceil}\left[\frac{Re((GD(x)))}{fibreweight} \right]$$



Once again I want to remind you to smoothen the transition where the layers end by not cutting them square. Now, how much will this board flex?

$$GD\_build(x) := GL(x) \cdot fibreweight$$

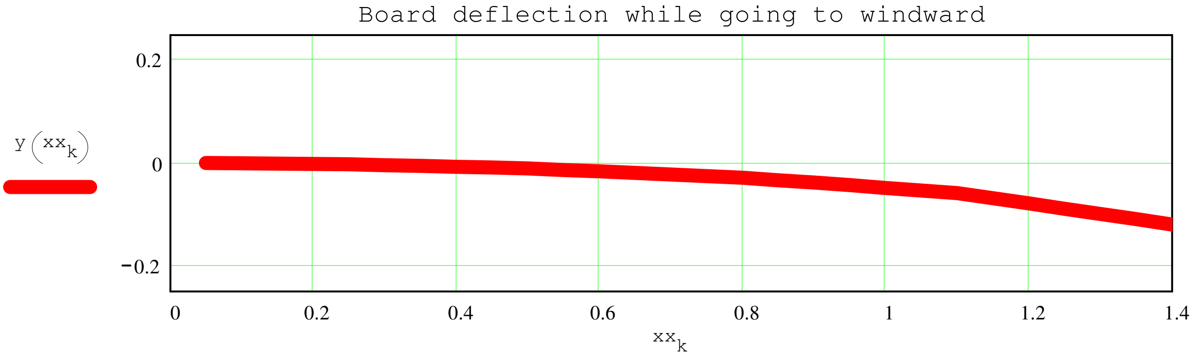
$$t\_build(x) := GD\_build(x) \cdot TGD$$

$$I\_build(x) := \frac{b \cdot \left[H^3 - (H - 2 \cdot t\_build(x))^3\right]}{12}$$

$$yd(x) := \frac{-1}{24} \cdot \frac{FRM}{E \cdot I\_build(x) \cdot l} \cdot \left[4 \cdot l^3 - (l - x)^3\right] + \frac{1}{8} \cdot FRM \cdot \frac{(l - x)^3}{E \cdot I\_build(x) \cdot l}$$

$$y(x) := \int_0^x \frac{-1}{24} \cdot \frac{FRM}{E \cdot I\_build(x) \cdot l} \cdot \left[4 \cdot l^3 - (l - x)^3\right] + \frac{1}{8} \cdot FRM \cdot \frac{(l - x)^3}{E \cdot I\_build(x) \cdot l} \, dx$$

$y(l - 0.001 \cdot m) = -0.12 \cdot m$ Stiffer!



Closing remark

Use cedar, balsa or very tough foam for the shear web. Another alternative is to build a composite I-beam or box-beam. Both Malcolm Tennant and John Shuttleworth use this type of design. Ian Farrier likes balsa. Not only are these materials tougher than a moderately dense PVC foam but they have much higher shear rigidity. I have neglected this aspect of the shear web in my calculations above but it has big impact on the over all stiffness of the boards.